CE 418.3 - Design in Reinforced Concrete

MIDTERM EXAMINATION

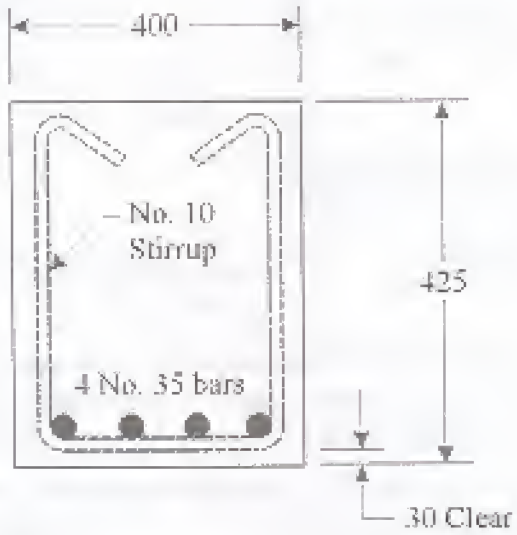
October 31, 2001

Time Allowed: 2 Hours | Professor: B. Sparling

Notes:
- Closed book examination
- CPCA Concrete Design Handbook may be used
- Calculators may be used
- The value of each question is provided along the left margin
- Supplemental material is provided at the end of the exam (i.e. formulas)
- Show **all** your work, including all formulas and calculations

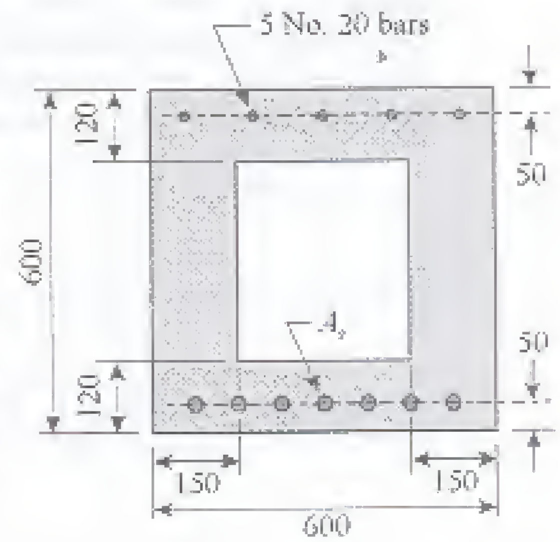
MARKS

20 **QUESTION 1:** The reinforced concrete beam shown below is constructed using concrete with $f_c' = 30$ MPa and Grade 400 reinforcing steel. Calculate the ultimate positive bending moment resistance M_r of the beam in accordance with the requirements of CSA A23.3-94 (i.e. using Whitney stress block, etc.).



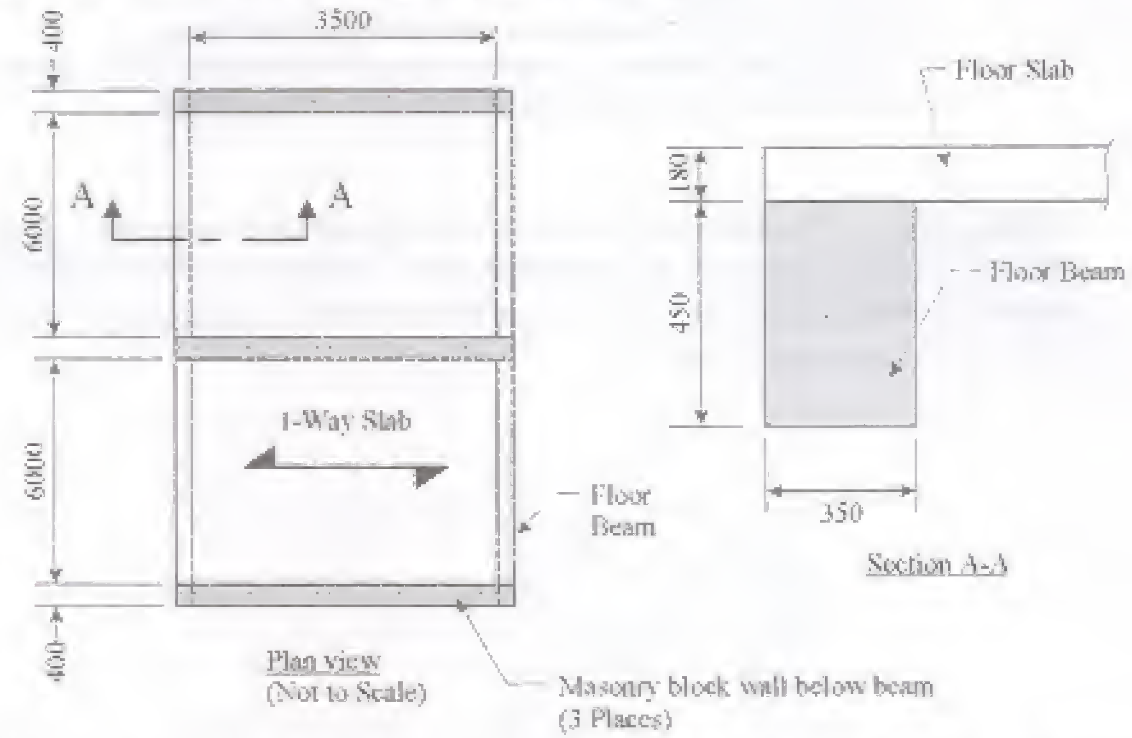
QUESTION 2: The reinforced concrete box girder (beam with a central rectangular void or hole) shown below is constructed using concrete with $f_c' = 25$ MPa and Grade 400 reinforcing steel.

20 **a)** If the tension reinforcing steel A_s consists of 7 No. 30 bars, calculate the ultimate positive bending moment resistance M_r of the box girder in accordance with the requirements of CSA A23.3-94. Assume that both the compression and tension reinforcing steel yields (no proof is required) and compensate for the effect of the holes in the concrete created by the compression steel. **Hint:** The compression block in the concrete extends below the depth of the top flange ($a > 120$ mm).



10 **b)** If, instead of 7 No. 30 bars as assumed in Part a), the tension reinforcing steel A_s was selected so that the depth of the compression block in the concrete was exactly equal to the depth of the top flange ($a = 120$ mm), would the compression reinforcing steel yield? Start from the basic principle of strain compatibility and include a sketch of the strain distribution over the height of the section.

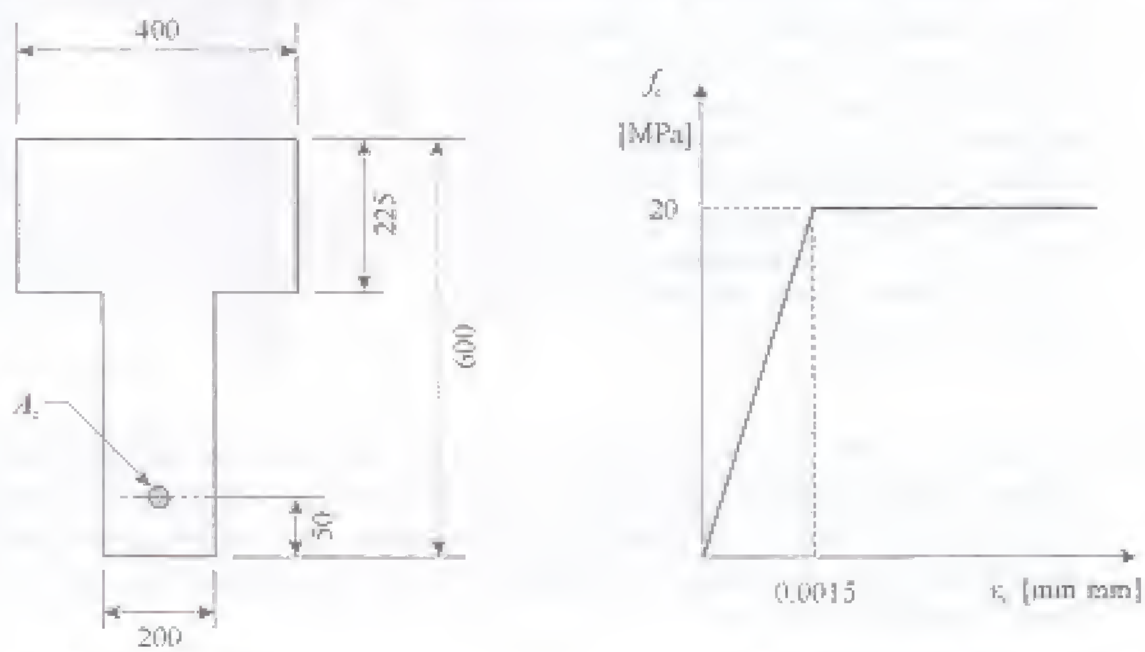
QUESTION 3 : The interior reinforced concrete floor system shown below consists of a one-way slab that is simply supported by floor beams which are poured separately (i.e. non-integral construction). The two-span continuous beams are simply supported on three masonry walls. The slab supports a specified dead load of $q_D = 3.6\ \text{kPa}$ and a live load of $q_L = 4.8\ \text{kPa}$, in addition to its own weight. Material properties are given by $f'_c = 30\ \text{MPa}$ and $f_y = 400\ \text{MPa}$. **Hint:** Design aids provided in the CPCA Concrete Design Handbook can be used to assist in answering the questions below.



12 **a)** Design the reinforcing steel for the critical **positive moments** in the floor slab, satisfying the requirements of CSA A23.3-94. Use the centre-to-centre distance between floor beams as the simple span length of the slab and assume a clear concrete cover of 20 mm on the main bars.

18 **b)** Design the reinforcing steel for the critical **negative moment** in the floor beam at the interior support according to the requirements of CSA A23.3-94. Assume a clear cover of 30 mm and that No. 10 stirrups are used.

QUESTION 4 : The T-beam shown in Part (i) of the figure below is fabricated using Grade 500 reinforcing steel and a new type of foam concrete that exhibits the stress-strain characteristics shown in Part (ii) of the figure below. Answer the following questions concerning the **nominal** response (i.e. ideal response with no resistance factors applied) of the beam, assuming ideal elasto-plastic behaviour in the reinforcing steel and negligible tensile strength in the concrete. The section is subjected to a positive bending moment.



(i) Beam cross-section (ii) Stress-strain curve: Foam concrete

QUESTION 4 : (continued)

12 a) Determine the theoretical area of reinforcing steel, A_s, that would cause the neutral axis to coincide with the bottom edge of the 400 mm wide top flange (225 mm below the top of the beam) just as the reinforcing steel reaches its yielding strain ($\varepsilon_s = \varepsilon_y$). Do not select the actual bar size or number of bars required.

8 b) Based on the theoretical area of reinforcing steel (A_s) found in Part a), calculate the corresponding nominal moment capacity of the beam.

Supplemental Material:

- **Material Properties:** $\phi_c = 0.6$ $\phi_s = 0.85$ $\alpha_D = 1.25$ $\alpha_L = 1.5$

$f'_{ct} = \dfrac{t}{\alpha + \beta t} f'_c$ $\dfrac{f_c}{f'_c} = 2\left(\dfrac{\varepsilon_c}{\varepsilon'_c}\right) - \left(\dfrac{\varepsilon_c}{\varepsilon'_c}\right)^2$ $f_{ct} = \dfrac{2P}{\pi d L} \approx 0.53\sqrt{f'_c}$

$E_c = \left(3300\sqrt{f'_c} + 6900\right)\left(\gamma_c/2300\right)^{1.5}$ $E_s = 200{,}000$ MPa $\varepsilon_{cu} = 0.0035$

$f_r = 0.6\,\lambda\sqrt{f'_c}$ $\gamma_c = 2400$ kg/m^3

- **Flexural Analysis:** $\Sigma F_x = 0$ $\Sigma M = 0 \rightarrow M = T\,(j\,d) = C_c\,(j\,d)$

$C_c = \int_0^c f_c\,dA$ $\bar{y}\,C_c = \int_0^c y\,f_c\,dA$ $C_c = (\phi_c\,\alpha_1\,f'_c)(\text{Area})$ $T = \phi_s\,A_s\,f_s$

$\alpha_1 = 0.85 - 0.0015\,f'_c \geq 0.67$ $\beta_1 = 0.97 - 0.0025\,f'_c \geq 0.67$ $a = \beta_1\,c$

$a = \dfrac{\phi_s\,A_s\,f_y}{\phi_c\,\alpha_1\,f'_c\,b}$ $\varepsilon_s = \varepsilon_{cu}\left(\dfrac{d-c}{c}\right)$ $\dfrac{c}{d} \leq \dfrac{700}{700 + f_y}$ $\dfrac{d'}{c} \leq 1 - \dfrac{f_y}{700}$

$(A_s)_{bal} = \dfrac{\phi_c\,\alpha_1\,f'_c\,\beta_1\,b\,d}{\phi_s\,f_y}\left(\dfrac{700}{700 + f_y}\right)$ $A_{s1} = A'_s\left(\dfrac{f'_s}{f_y} - \dfrac{\phi_c\,\alpha_1\,f'_c}{\phi_s\,f_y}\right)$ $A_{s2} = A_s - A_{s1}$

$M_{r1} = \phi_s\,A_{s1}\,f_{s1}\,(d - d')$ $M_{r2} = \phi_s\,A_{s2}\,f_{s2}\left(d - \dfrac{a}{2}\right)$ $\varepsilon'_s = \varepsilon_{cu}\left(\dfrac{c - d'}{c}\right)$

- **Flexural Design:** $A_{s_{min}} = \dfrac{0.2\sqrt{f'_c}}{f_y}\,b_t\,h$ $\rho = \dfrac{A_s}{b\,d}$ $K_r = \dfrac{M_r \times 10^6}{b\,d^2}$

$\rho_{bal} = \dfrac{\phi_c\,\alpha_1\,f'_c\,\beta_1}{\phi_s\,f_y}\left(\dfrac{700}{700 + f_y}\right)$ $K_r = \phi_s\,\rho\,f_y\left(1 - \dfrac{\phi_s\,\rho\,f_y}{2\,\phi_c\,\alpha_1\,f'_c}\right)$ $M_r \geq M_f$

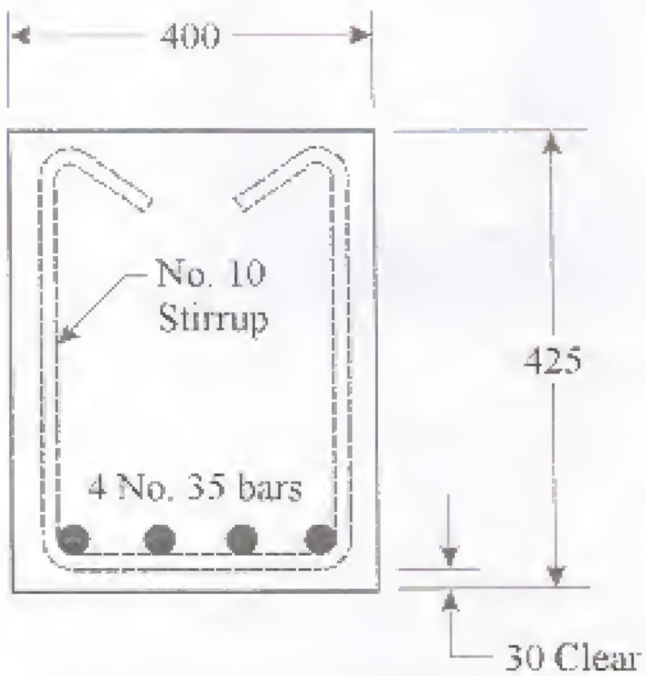
$M_r = \phi_s\,\rho\,f_y\left(1 - \dfrac{\phi_s\,\rho\,f_y}{2\,\phi_c\,\alpha_1\,f'_c}\right)b\,d^2$ $\rho = \dfrac{\phi_c\,\alpha_1\,f'_c \pm \sqrt{(\phi_c\,\alpha_1\,f'_c)^2 - 2\,K_r\,\phi_c\,\alpha_1\,f'_c}}{\phi_s\,f_y}$

- **One-Way Floor Systems:** $A_{s_{min}} = 0.002\,A_g$ $A_{sf} = \dfrac{(\phi_c\,\alpha_1\,f'_c)(b_f - b)}{\phi_s\,f_y}$

Basic Parameters: $E_s := 200000 \cdot MPa$ $\phi_s := 0.85$ $\phi_c := 0.60$ $\varepsilon_{cu} := 0.0035$

$\alpha_D := 1.25$ $\alpha_L := 1.5$ $\gamma_c := 2400 \cdot \frac{kg}{m^3}$

Question 1:



Given:

$f'_c := 30 \cdot MPa$ $f_y := 400 \cdot MPa$

$h := 425 \cdot mm$ $b := 400 \cdot mm$

$d_b := 35 \cdot mm$ $A_{bar} := 1000 \cdot mm^2$ $n_{bar} := 4$

$cc := 30 \cdot mm$ $A_s := n_{bar} \cdot A_{bar}$

- Whitney stress block parameters:

$\alpha_1 := 0.85 - 0.0015 \cdot \frac{f'_c}{MPa}$ $\alpha_1 = 0.805$

$\beta_1 := 0.97 - 0.0025 \cdot \frac{f'_c}{MPa}$ $\beta_1 = 0.895$

- Effective beam depth: $d := h - \left(cc + 10 \cdot mm + \frac{d_b}{2}\right)$ $d = 367.5\,mm$

- Check if under or over-reinforced:

$$A_{sb} := \frac{\phi_c \cdot \alpha_1 \cdot f'_c \cdot \beta_1 \cdot b \cdot d}{\phi_s \cdot f_y} \cdot \left(\frac{700 \cdot MPa}{700 \cdot MPa + f_y}\right) \qquad A_{sb} = 3568.08501\,mm^2$$

$\frac{A_s}{A_{sb}} = 1.12105$ <--- Therefore, section is over-reinforced and steel will not yield

- Strain and stress in reinforcing steel: $\varepsilon_s = \varepsilon_{cu} \cdot \left(\frac{d-c}{c}\right)$ $f_s = E_s \cdot \varepsilon_s = E_s \cdot \left[\varepsilon_{cu} \cdot \left(\frac{d-c}{c}\right)\right]$

- Depth of compression stress block: $\Sigma F_x = 0$ $C_c = T$

$$C_c = (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (a \cdot b) = (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (\beta_1 \cdot c \cdot b)$$

$$T = \phi_s \cdot A_s \cdot f_s = \phi_s \cdot A_s \cdot \left[E_s \cdot \left[\varepsilon_{cu} \cdot \left(\frac{d-c}{c}\right)\right]\right]$$

$$(\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (\beta_1 \cdot c \cdot b) = \phi_s \cdot A_s \cdot \left[E_s \cdot \left[\varepsilon_{cu} \cdot \left(\frac{d-c}{c}\right)\right]\right]$$

$$(\phi_c \cdot \alpha_1 \cdot f'_c \cdot \beta_1 \cdot b) \cdot c^2 + (\phi_s \cdot A_s \cdot E_s \cdot \varepsilon_{cu}) \cdot c - \phi_s \cdot A_s \cdot E_s \cdot \varepsilon_{cu} \cdot d = 0$$

where: $(\phi_c \cdot \alpha_1 \cdot f'_c \cdot \beta_1 \cdot b) = 0.00519\,\mathrm{m}^2 \dfrac{MPa}{mm}$ $\quad (\phi_s \cdot A_s \cdot E_s \cdot \varepsilon_{cu}) = 2.38 \times 10^6\,\mathrm{N}$

$$\phi_s \cdot A_s \cdot E_s \cdot \varepsilon_{cu} \cdot d = 8.7465 \times 10^8\, N \cdot mm$$

- Solving:

$$c := \frac{1}{(2 \cdot \phi_c \cdot \alpha_1 \cdot f'_c \cdot \beta_1 \cdot b)} \cdot \left[-\phi_s \cdot A_s \cdot E_s \cdot \varepsilon_{cu} \ldots + \left(\phi_s^2 \cdot A_s^2 \cdot E_s^2 \cdot \varepsilon_{cu}^2 + 4 \cdot \phi_c \cdot \alpha_1 \cdot f'_c \cdot \beta_1 \cdot b \cdot \phi_s \cdot A_s \cdot E_s \cdot \varepsilon_{cu} \cdot d\right)^{\left(\frac{1}{2}\right)} \right]$$

$c = 240.95481\, mm$ $\quad a := \beta_1 \cdot c$ $\quad a = 215.65455\, mm$

- Check equilibrium: $\varepsilon_s := \varepsilon_{cu} \left(\dfrac{d - c}{c}\right)$ $\quad \varepsilon_s = 0.00184$ $\quad \varepsilon_y := \dfrac{f_y}{E_s}$ $\quad \dfrac{\varepsilon_s}{\varepsilon_y} = 0.91907$

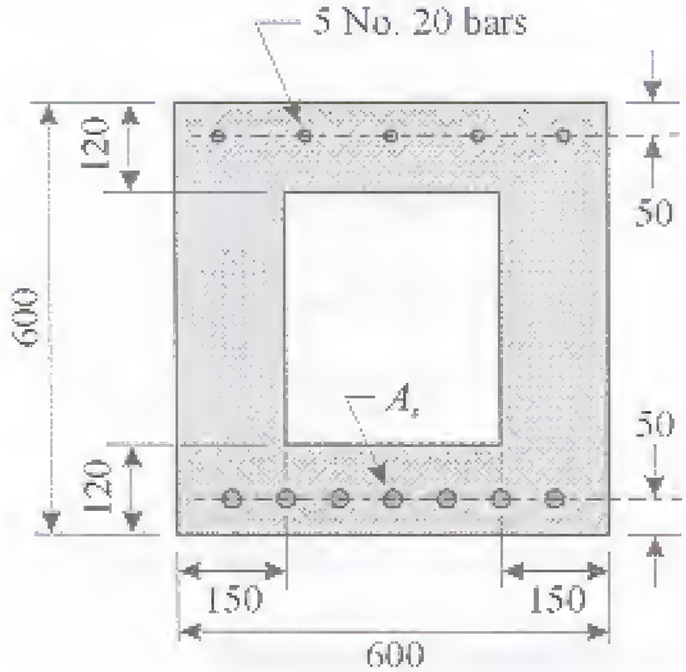
$f_s := \varepsilon_s \cdot E_s$ $\quad f_s = 367.62759\, MPa$ $\quad T := \phi_s \cdot A_s \cdot f_s$ $\quad T = 1249.93379\, kN$

$C_c := (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (a \cdot b)$ $\quad C_c = 1249.93379\, kN$ $\quad$ - OK

- Ultimate moment resistance:

$M_r := T \cdot \left(d - \dfrac{a}{2}\right)$ $\quad M_r = 324.57371\, kN \cdot m$

Question 2:



Given: $f'_c := 25 \cdot MPa$ $\quad f_y := 400 \cdot MPa$

$d'_b := 20 \cdot mm$ $\quad A'_{bar} := 300 \cdot mm^2$

$n'_{bar} := 5$ $\quad A'_s := n'_{bar} \cdot A'_{bar}$

$A'_s = 1500\,mm^2$

$b_{ff} := 600 \cdot mm$ $\quad h_{ff} := 120 \cdot mm$ $\quad cc := 50 \cdot mm$

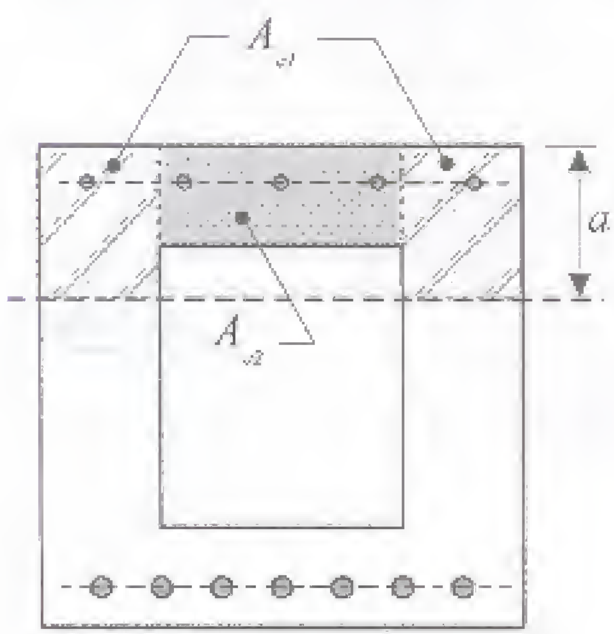
$b_w := 150 \cdot mm$ $\quad h := 600 \cdot mm$

Solution: $d := h - cc$ $\quad d = 0.55\,m$ $\quad d' := cc$ $\quad d' = 50\,mm$

- Whitney stress block parameters:

$$\alpha_1 := 0.85 - 0.0015 \cdot \frac{f'_c}{MPa} \quad \alpha_1 = 0.8125 \quad \beta_1 := 0.97 - 0.0025 \cdot \frac{f'_c}{MPa} \quad \beta_1 = 0.9075$$

Part (a) $\quad d_b := 30 \cdot mm$ $\quad A_{bar} := 700 \cdot mm^2$ $\quad n_{bar} := 7$ $\quad A_s := n_{bar} \cdot A_{bar}$ $\quad A_s = 4900\,mm^2$



- Portion of A_s balancing compression in flange:

$$C_{c2} := (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot [h_{ff} \cdot (b_{ff} - 2 \cdot b_w)]$$

$C_{c2} = 438.75\,kN$ $\quad \phi_s \cdot f_y \cdot A_{s1} = C_{c2}$

$$A_{s1} := \frac{C_{c2}}{(\phi_s \cdot f_y)} \qquad A_{s1} = 1290.44118\,mm^2$$

- Portion of A_s balancing compression steel:

$$C_s := \phi_s \cdot f_y \cdot A'_s - \phi_c \cdot \alpha_1 \cdot f'_c \cdot A'_s$$

$C_s = 491.71875\,kN$ $\quad \phi_s \cdot f_y \cdot A_{s2} = C_s$

$$A_{s2} := \frac{C_s}{(\phi_s \cdot f_y)} \qquad A_{s2} = 1446.23162\,mm^2$$

- Remaining tension steel area to balance compression in both webs:

$A_{s3} := A_s - A_{s1} - A_{s2}$ $\qquad A_{s3} = 2163.32721\,mm^2$

$(\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (2 \cdot b_w \cdot a) = \phi_s \cdot f_y \cdot A_{s3}$

$a := \frac{1}{2} \cdot \phi_s \cdot f_y \cdot \frac{A_{s3}}{(\phi_c \cdot \alpha_1 \cdot f'_c \cdot b_w)}$ $\qquad a = 201.17094\,mm$ $\quad$ > 120 mm - OK

$c := \frac{a}{\beta_1}$ $\qquad c = 221.67597\,mm$

- Moment capacity:

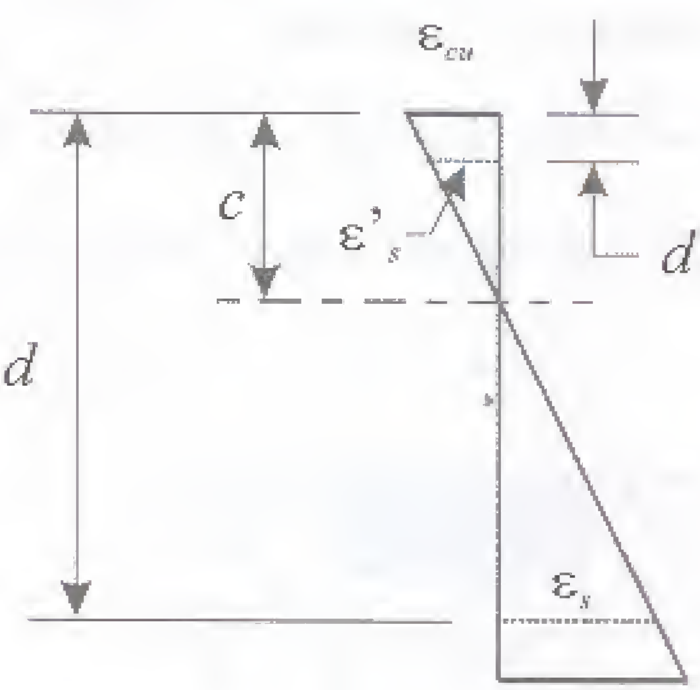
$M_{r1} := \phi_s \cdot f_y \cdot A_{s1} \cdot \left(d - \frac{h_F}{2}\right)$ $\qquad M_{r1} = 214.9875\,kN \cdot m$ <-- Flange

$M_{r2} := \phi_s \cdot f_y \cdot A_{s2} \cdot (d - d')$ $\qquad M_{r2} = 245.85937\,kN \cdot m$ <-- Compression steel

$M_{r3} := \phi_s \cdot f_y \cdot A_{s3} \cdot \left(d - \frac{a}{2}\right)$ $\qquad M_{r3} = 330.55843\,kN \cdot m$ <-- 2 Webs

$M_r := M_{r1} + M_{r2} + M_{r3}$ $\qquad M_r = 791.40531\,kN \cdot m$

Part b) $\quad a := 120 \cdot mm$ $\qquad \beta_1 = 0.9075$



$c := \frac{a}{\beta_1}$ $\qquad c = 132.2314\,mm$

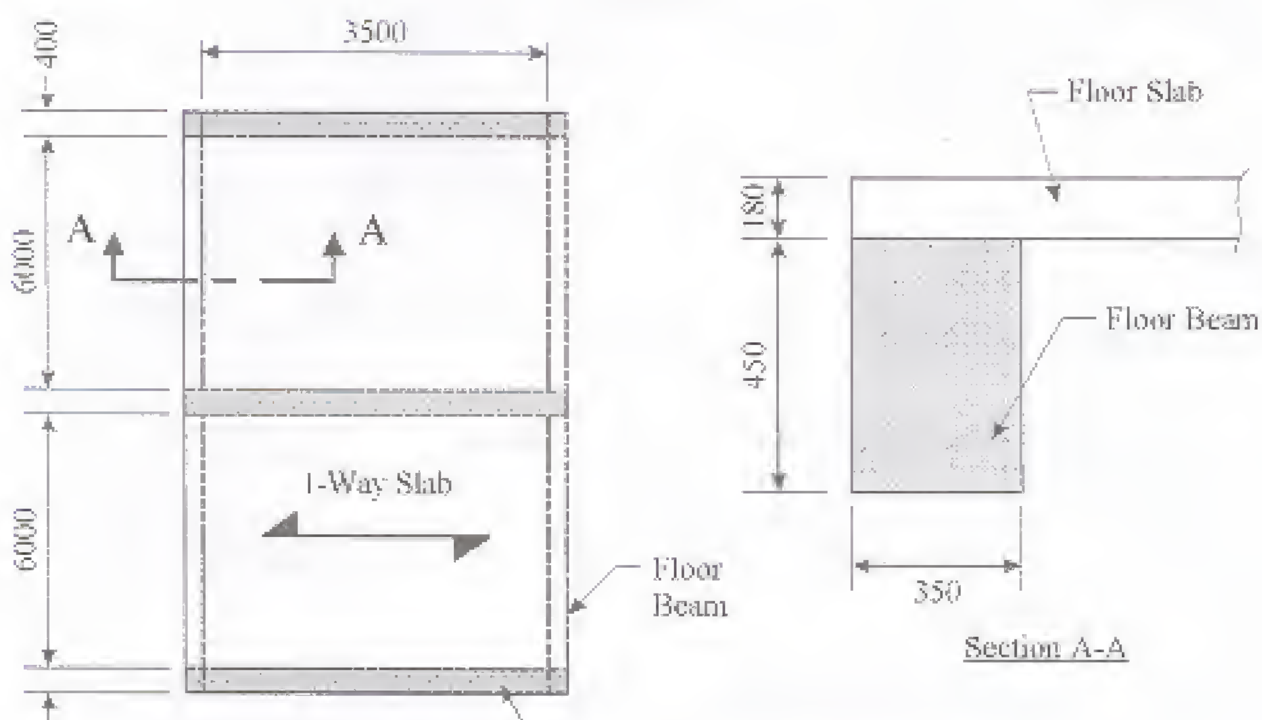
$\varepsilon'_s := \varepsilon_{cu} \cdot \left(\frac{c - d'}{c}\right)$ $\qquad \varepsilon'_s = 0.00218$

$\varepsilon_y = 0.002$ $\qquad \frac{\varepsilon'_s}{\varepsilon_y} = 1.08828$

Therefore, A'_s does yield.

$f_s := E_s \cdot \varepsilon'_s$ $\qquad f_s = 435.3125\,MPa$

Question 3:



Plan view
(Not to Scale)

Section A-A

Given:

- Materials: $f'_c := 30 \cdot MPa$ $\quad \varepsilon_{cu} := 0.0035$ $\quad f_y := 400 \cdot MPa$ $\quad E_s := 200000 \cdot MPa$

$\phi_c := 0.6$ $\quad \phi_s := 0.85$ $\quad \varepsilon_y := \dfrac{f_y}{E_s}$ $\quad \varepsilon_y = 0.002$

- Dimensions: $l_{ns} := 3.5 \cdot m$ $\quad l_{nb} := 6 \cdot m$ $\quad b_b := 350 \cdot mm$ $\quad h_b := 450 \cdot mm$

$h_s := 180 \cdot mm$

- Loads: $q_D := 3.6 \cdot kPa$ $\quad q_L := 4.8 \cdot kPa$ $\quad \gamma_c := 2400 \cdot \dfrac{kg}{m^3}$

$\alpha_D := 1.25$ $\quad \alpha_L := 1.5$

Basic Parameters:

$$\alpha_1 := \begin{cases} 0.85 - 0.0015 \cdot \dfrac{f'_c}{MPa} & \text{if } 0.85 - 0.0015 \cdot \dfrac{f'_c}{MPa} \ge 0.67 \\ 0.67 & \text{otherwise} \end{cases} \qquad \alpha_1 = 0.805$$

$$\beta_1 := \begin{cases} 0.97 - 0.0025 \cdot \dfrac{f'_c}{MPa} & \text{if } 0.97 - 0.0025 \cdot \dfrac{f'_c}{MPa} \ge 0.67 \\ 0.67 & \text{otherwise} \end{cases} \qquad \beta_1 = 0.895$$

- Balanced Reinforcement Ratio:

$$\rho_b := \frac{\phi_c \cdot \alpha_1 \cdot f_c \cdot \beta_1}{\phi_s \cdot f_y} \cdot \left(\frac{700}{700 + \frac{f_y}{MPa}} \right) \qquad \rho_b = 0.02427$$

Solution:

Part (a): Slab design

- Effective depth, d: Assume No. 15 bars $\quad d_b := 15 \cdot mm$

- Clear cover - Interior (non-corrosive) exposure: $\quad cc := 20 \cdot mm$

$$d := h_s - cc - \frac{d_b}{2} \qquad d = 152.5\,mm$$

- Unit design strip: $\quad b_s := 1000 \cdot mm$

- Slab self weight: $\quad w_{Dsw} := (b_s \cdot h_s) \cdot (\gamma_c \cdot g) \qquad w_{Dsw} = 4.23647 \frac{kN}{m}$

- Area loads: $\quad w_D := q_D \cdot b_s \qquad w_D = 3.6 \frac{kN}{m}$ (per m slab width)

$w_L := q_L \cdot b_s \qquad w_L = 4.8 \frac{kN}{m}$ (per m slab width)

- Factored loading:

$$w_{Df} := \alpha_D \cdot (w_{Dsw} + w_D) \qquad w_{Df} = 9.79559 \frac{kN}{m}$$

$$w_{Lf} := \alpha_L \cdot w_L \qquad w_{Lf} = 7.2 \frac{kN}{m}$$

$$w_f := w_{Df} + w_{Lf} \qquad w_f = 16.99559 \frac{kN}{m}$$

- Positive design moment (at midspan): $\quad L_s := l_{ns} + b_b \qquad L_s = 3850\,mm$

$$M_{pos} := \frac{w_f \cdot L_s^2}{8} \qquad M_{pos} = 31.48964\,kN \cdot m$$

- Using Normalised Moments and reinforcement ratios (Table 2.1):

$$K_r = \phi_s \cdot \rho \cdot f_y \cdot \left(1 - \frac{\phi_s \cdot \rho \cdot f_y}{2 \cdot \phi_c \cdot \alpha_1 \cdot f_c}\right)$$

$$\rho(K_r) := \frac{\left[\phi_c \cdot \alpha_1 \cdot f_c - \left(\phi_c^2 \cdot \alpha_1^2 \cdot f_c^2 - 2 \cdot K_r \cdot \phi_c \cdot \alpha_1 \cdot f_c\right)^{\left(\frac{1}{2}\right)}\right]}{(f_y \cdot \phi_s)}$$

$M_r := M_{pos}$ $\qquad K_r := \dfrac{M_r}{b_s \cdot d^2}$ $\qquad K_r = 1.35403\,MPa$

Therefore: $\quad \rho(K_r) = 0.00419$

$A_{s\_req} := \rho(K_r) \cdot b_s \cdot d$ $\qquad A_{s\_req} = 638.7063\,mm^2$

- Check minimum steel area: $\quad A_{min} := 0.002 \cdot (b_s \cdot h_s)$ $\qquad A_{min} = 360\,mm^2$
(temp. and shrinkage)

$$A_s := \begin{cases} A_{s\_req} & \text{if } A_{s\_req} \ge A_{min} \\ A_{min} & \text{otherwise} \end{cases} \qquad A_s = 638.7063\,mm^2$$

Try using No. 10 bars. $\qquad A_{s\_bar} := 100 \cdot mm^2 \qquad d_b := 10 \cdot mm$

- Number of bars: $\quad n_b := \dfrac{A_s}{A_{s\_bar}}$ $\qquad n_b = 6.38706$ bars per m width

- Required bar spacing: $\quad s_{b\_req} := \dfrac{b_s}{n_b}$ $\qquad s_{b\_req} = 156.56648\,mm$

- Check maximum spacing:

$$s_{max} := \begin{cases} 3 \cdot h_s & \text{if } 3 \cdot h_s \le 500 \cdot mm \\ 500 \cdot mm & \text{otherwise} \end{cases} \qquad s_{max} = 500\,mm$$

Therefore:

$$s := \begin{cases} s_{b\_req} & \text{if } s_{b\_req} \le s_{max} \\ s_{max} & \text{otherwise} \end{cases} \qquad s = 156.56648\,mm$$

Choose: $\quad s := 150 \cdot mm \qquad n_b := \dfrac{b_s}{s} \qquad n_b = 6.66667$ bars per m

$A_s := n_b \cdot A_{s\_bar}$ $\qquad A_s = 666.66667\,mm^2$

$\rho_{act} := \dfrac{A_s}{b_s \cdot d}$ $\qquad \rho_{act} = 0.00437$ $\qquad \dfrac{\rho_{act}}{\rho_b} = 0.1801$ $\qquad$ <-- Steel yields - OK

Say use No. 10 @ 150 mm o.c. for positive slab reinforcement

Part (b): Design of Floor Beam

- Beam self weight. $w_{Dsw} := (b_b \cdot h_b) \cdot (\gamma_c \cdot g)$ $\qquad w_{Dsw} = 3.70691 \frac{kN}{m}$

- Loading on floor beam from slab. Slab reaction force

$R_{slab} := \frac{l}{2} w_f \cdot L_s \qquad R_{slab} = 32.72\, kN$ per m width

Therefore, the uniformly distributed line load on the interior floor beam (incl. self weight) is:

$$w_{fb} := \frac{R_{slab}}{b_s} + \alpha_D \cdot w_{Dsw} \qquad w_{fb} = 37.35 \frac{kN}{m}$$

-Effective depth. Assume No. 30 bars and include a No. 10 stirrup $\qquad d_b := 30 \cdot mm$

- Clear cover: $cc := 30 \cdot mm$

$d := h_b - cc - 10 \cdot mm - 0.5 \cdot d_b \qquad d = 395\, mm$

- Negative support moments: - Design moments. Cl. 9.3.3

- End spans - Discontinuous end unrestrained:

$$M_{neg} := \frac{-w_{fb} \cdot l_{nb}^{\,2}}{9} \qquad M_{neg} = -149.40062\, kN \cdot m \qquad M_r := -M_{neg}$$

$$K_r := \frac{M_r}{b_b \cdot d^2} \qquad K_r = 2.73584\, MPa$$

Therefore: $\rho(K_r) = 0.009$

$A_{s\_req} := \rho(K_r) \cdot b_b \cdot d \qquad A_{s\_req} = 1243.70478\, mm^2$

Try No. No. 25 bars: $d_b := 25 \cdot mm \qquad n_b := 3 \qquad A_{s\_bar} := 500 \cdot mm^2$

$A_s := n_b \cdot A_{s\_bar} \qquad A_s = 1500\, mm^2 \qquad > A_{s\_req}$ - OK

Bar spacing: $agg := 20 \cdot mm$ - Aggregate size

$$s := \left| \begin{array}{l} s_1 \leftarrow 1.4 \cdot d_b \\ s_2 \leftarrow 1.4 \cdot agg \\ s_3 \leftarrow s_1 \ \ if\ \ s_1 \ge s_2 \\ s_3 \leftarrow s_2 \ \ otherwise \\ s_4 \leftarrow s_3 \ \ if\ \ s_3 \ge 30 \cdot mm \\ s_4 \leftarrow 30 \cdot mm \ \ otherwise \\ s_4 \end{array} \right. \qquad s = 35\, mm$$

Check beam web width: $b_{min} := 2\cdot(cc + 10\cdot mm) + n_b\cdot d_b + (n_b - 1)\cdot s$

$b_{min} = 0.225\,m$ $< b_w = 350$ mm - OK

Check yielding:

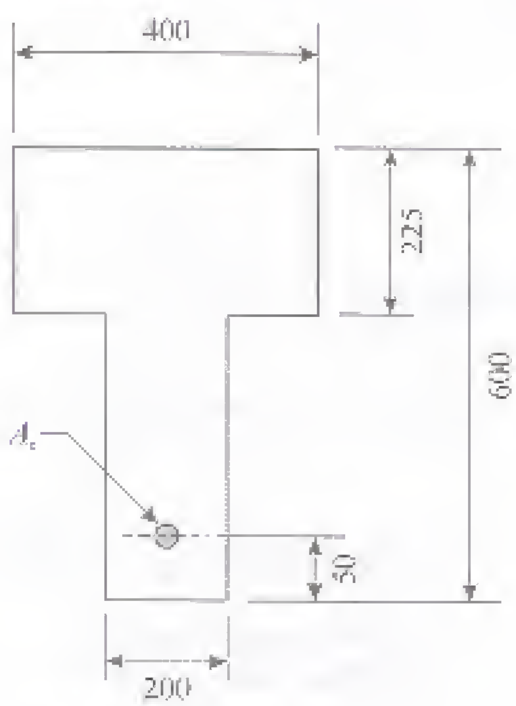
$A_s = 1500\,mm^2$ $\rho_{act} := \dfrac{A_s}{b_b\cdot d}$ $\rho_{act} = 0.01085$ $\dfrac{\rho_{act}}{\rho_b} = 0.447$

Check minimum reinforcement:

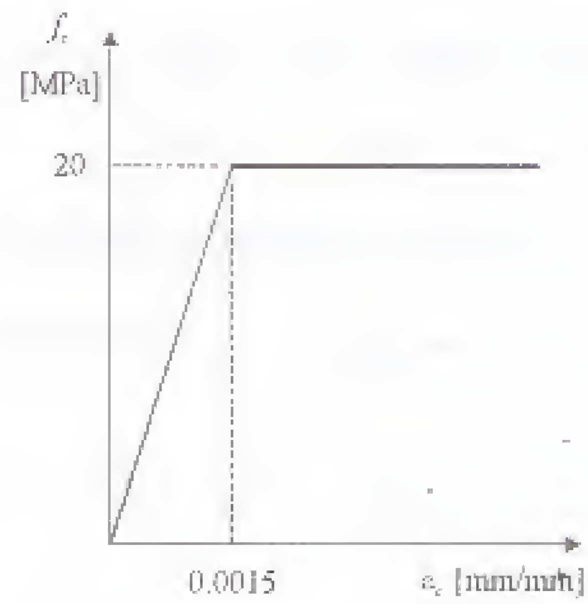
$$A_{smin} := 0.2\cdot MPa\cdot\frac{\sqrt{\dfrac{f_c}{MPa}}}{f_y}\cdot b_b\cdot h_b \qquad A_{smin} = 431.33151\,mm^2 \quad \text{- OK}$$

Therefore, use 3 No. 25 bars for negative reinforcement in interior floor beam

Question 4:



(i) Beam cross-section



(ii) Stress-strain curve: Foam concrete

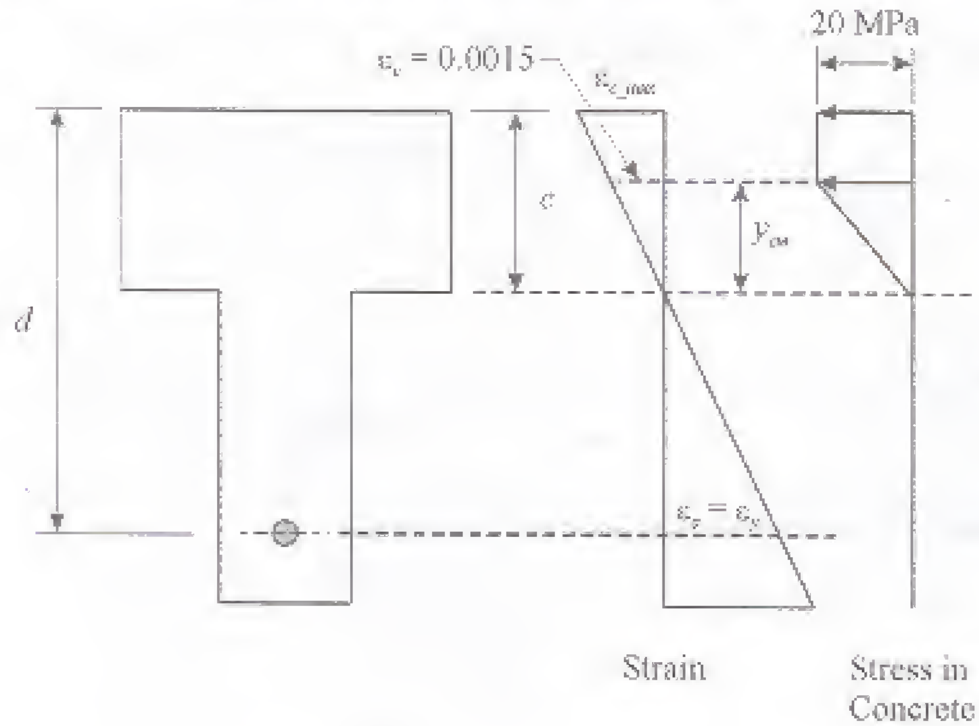
Given:

$b_F := 400\cdot mm$ $h_F := 225\cdot mm$ $f_y := 500\cdot MPa$ $E_s := 200000\cdot MPa$

$d := 550\cdot mm$ $\varepsilon_y := \dfrac{f_y}{E_s}$ $\varepsilon_y = 0.0025$

$\varepsilon_{cm} := 0.0015$ $f_{cm} := 20\cdot MPa$ $E_c := \dfrac{f_{cm}}{\varepsilon_{cm}}$ $E_c = 13333.33333\,MPa$

a) Steel area for neutral axis at bottom of flange when steel yields



- Maximum strain at top fibre:

$$\varepsilon_{c\_max} := \varepsilon_y \cdot \left(\frac{h_F}{d - h_F}\right)$$

$$\varepsilon_{c\_max} = 0.00173$$

- Height at start of "yield stress" in foam concrete:

$$y_{cm} := h_F \cdot \frac{\varepsilon_{cm}}{\varepsilon_{c\_max}}$$

$$y_{cm} = 195\, mm$$

- Compressive force in "yielded" portion of flange: (above y_{cm})

$$C_{c1} := f_{cm} \cdot b_F \cdot (h_F - y_{cm}) \qquad C_{c1} = 240\, kN$$

- Compression force in linearly varying concrete stress region (flange below y_{cm})

- Strain: $\varepsilon_c = \varepsilon_{c\_max} \cdot \dfrac{y}{h_F}$ - Concrete stress: $f_{c\_lin}(y) := \left(\varepsilon_{c\_max} \cdot \dfrac{y}{h_F}\right) \cdot E_c$

$$C_{c2} := \int_0^{y_{cm}} f_{c\_lin}(y) \cdot b_F \, dy \qquad C_{c2} = 780\, kN \qquad OR \qquad (0.5 \cdot f_{cm}) \cdot (b_F \cdot y_{cm}) = 780\, kN$$

(Linear stress distribution: Average stress x area)

$$C_c := C_{c1} + C_{c2} \qquad C_c = 1020\, kN$$

- Required steel area to balance this compression, knowing that the steel yields (given)

$$\Sigma F_x = 0 \qquad T = C_c \qquad A_s \cdot f_y = C_c$$

$$A_s := \frac{C_c}{f_y} \qquad A_s = 2040\, mm^2$$

b) Nominal moment capacity

- Compression in "yielded" portion of flange:

- Moment arm $jd_1 := d - \dfrac{(h_F - y_{cm})}{2}$ $jd_1 = 535\, mm$

- Moment $M_{C1} := C_{c1} \cdot jd_1$ $M_{C1} = 128.4\, kN \cdot m$

- Compression in linearly varying stress region

$$y_{bar} := \frac{\int_0^{y_{cm}} y \cdot f_{c\_lin}(y) \cdot b_{fr}\, dy}{C_{c2}} \qquad y_{bar} = 130\,mm \qquad \text{OR} \qquad \frac{2}{3} \cdot y_{cm} = 130\,mm$$

- Moment arm $\quad jd_2 := d - h_{fr} + y_{bar} \qquad jd_2 = 455\,mm$

- Moment $\quad M_{C2} := C_{c2} \cdot jd_2 \qquad M_{C2} = 354.9\,kN \cdot m$

- Total nominal moment capacity :

$$M_n := M_{C1} + M_{C2} \qquad M_n = 483.3\,kN \cdot m$$